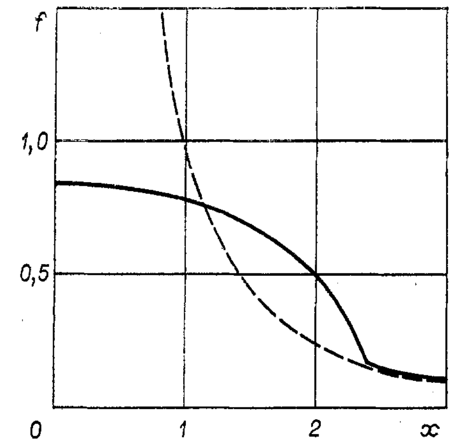

Fig. 1

while for the point liberation of energy we have $r_f \sim t^{1/11}$ and $dr_f/dt \sim t^{-10/11}$.

Thus, we have shown that the initial spatial distribution of released energy has an appreciable influence on the propagation of the thermal wave.

In conclusion, we note that later stages in the process, when the velocity of the thermal wave front is comparable with that of sound in the heated gas, can be described by the methods examined in [1, 2].

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RADIATIVE—CONDUCTIVE HEAT TRANSFER IN A THIN SEMITRANSPARENT PLATE IN THE GUIDED-WAVE APPROXIMATION FOR A TEMPERATURE- AND FREQUENCY-DEPENDENT ABSORPTION COEFFICIENT

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UDC 536.2

The radiative—conductive heat-transfer problem has been studied previously [1] for a thin semitransparent cylinder whose refractive index is much greater than unity. This class of problems arises in the investigation of the temperature fields in semitransparent crystals such as sapphire or lithium niobate during pulling from the melt by the Czochralski or Stepanov method. It is shown that the radiative energy transfer in the indicated cylinder can be described by the so-called guided-wave approximation, where only those rays which undergo total internal reflection at the boundary of the cylinder are included in the radiant flux in its interior. A comparison of the analytical with experimental results and a study of heat transfer in a semitransparent cylinder coated with a thin absorbing film are reported in [2]. Heat transfer in a thin infinitely wide semitransparent plate is discussed in the same paper. However, the constant absorption coefficient postulated in [1, 2] appears to be rather crude. For example, according to the data of [3], the absorption coefficient of sapphire in the temperature range 1200-2000°C varies quite considerably, from 0.004 to 0.5 cm^{-1}, in the interval of wavelengths up to 4 μm, where the bulk of the radiated energy is concentrated. It is important, therefore, to calculate the temperature field in a semitransparent plate whose absorption coefficient depends on the temperature and frequency.

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Generally speaking, the modeling of processes of crystal pulling from the melt requires solution of the more general heat-transfer problem in the melt and solid phase, with a simultaneous determination of the position and geometry of the crystallization front. However, the influence of the front on the temperature field in a thin plate extends only a small distance of the order of its thickness. The heat-transfer problem can therefore be separated into two problems: calculation of the main temperature field in the crystal and calculation of the correction induced by the influence of the interface geometry and position. Moreover, inasmuch as the pulling rate of high-melting crystals is small, its influence is usually neglected. As a result, for the steady-state pulling process we arrive at an independent heat-transfer problem in a thin semitransparent plate, and this problem is the subject of the present study.

Thus, let plane shields be set up parallel to the plate on both sides, serving as secondary heaters. The temperature of the shields is assumed to be known and constant over their width. The gap between the shields and the plate is evacuated or filled with a gas with negligible absorption, and free convection can be neglected. We also assume that scattering is absent, the side surface of the plate is transparent and perfectly smooth, and the thermal conductivity is isotropic. We consider the ends of the plate to be absolutely black. In this case the warmer base corresponds to the crystallization front in the pulling of a crystal from the melt. Let the following geometrical relations also be satisfied in the system:

$$d/Z \ll 1,\ d/b \ll 1,\ h/Z \ll 1,\ k_\lambda d \ll 1, \tag{1}$$

where d, b, and Z are the thickness, width, and length of the plate; h, distance between the plate and the shield; and k_λ, absorption coefficient at wavelength λ. Then, as a result of the stated assumptions, the thermal radiation in the plate will undergo multiple reflection and refraction at its side surface. The radiant flux will therefore essentially contain only the part which propagates along the plate as in an optical waveguide. Formally, this situation is equivalent to replacement of the real reflection coefficient by a step function:

$$R(\Theta) = \begin{cases} 1 & \text{for} \quad \Theta \geqslant \Theta_t, \\ 0 & \text{for} \quad \Theta < \Theta_t, \end{cases}$$

where Θ_t is the angle of total internal reflection.

It is essential to note that light rays can be reflected not only at the wide sides, but also at the narrow sides of the plate. The guided-wave approximation therefore holds if

$$k_\lambda b \gg 1 \quad \text{or} \quad k_\lambda b \ll 1,\ b/Z \ll 1. \tag{2}$$

In the first case the reflections from the narrow faces of the plate can be neglected, corresponding to an infinitely wide plate from the radiative-transfer point of view, while in the second case there are quite a few of the indicated reflections, and the problem is essentially reducible to heat transfer in a cylinder with a rectangular base (bar).

In both cases, imposing the above-stated constraints (1) and (2), we can assume that the temperature field varies only along the length of the plate. Then the radiative–conductive heat-transfer equation is written in the one-dimensional form

$$\frac{d}{dx}\left(\varkappa(T)\frac{dT}{dx}\right) = \frac{1}{bd}\int\limits_S\int\limits_\Omega\int\limits_\lambda k_\lambda(T)\left(B_\lambda(T) - i_\lambda(x, y, z, \Omega)\right) d\lambda d\Omega dy dz, \tag{3}$$

where $T = T_0$ at $x = 0$ and $T = T_1$ at $x = Z$. Here $B_\lambda(T)$ is the spectral radiation density of an absolute black body:

$$B_\lambda(T) = 2hc^2/(\lambda^5 n^2(\exp(hc/n\lambda kT) - 1)),$$

$$\int B_\lambda(T)\, d\lambda = \frac{\sigma}{\pi} n^2 T^4,$$

where n is the refractive index, σ is the Stefan–Boltzmann constant, and i_λ is the radiation intensity arriving at the point (x, y, z). To determine i_λ we use the equation

$$di_\lambda/ds + k_\lambda i_\lambda = k_\lambda B_\lambda(T),$$

in which s is the path length along the light ray.

TABLE 1

T, °C \ λ, μm	0,25—0,5	0,5—1	1—3	3—4	4—5
1200	0,06	0,02	0,005	0,13	2,0
1500	0,2	0,1	0,027	0,14	2,4
1600	0,3	0,2	0,07	0,16	2,7
1700	0,6	0,35	0,12	0,18	3,0
2020	0,4	0,27	0,14	0,30	4,0

The radiation intensity is made up of the radiation from the bases of the plate i_{f1}, i_{f2}, the plate proper i_c, and the shields i_s. Accordingly, for the spatial radiation density we have

$$I_\lambda = \int i_\lambda d\Omega = I_{f1} + I_{f2} + I_c + I_s. \tag{4}$$

As mentioned previously, in the calculation of I_λ it is only required to include those rays whose angle of incidence on the side surface of the plate is greater than Θ_t. We first consider the term I_{f1}, which describes the radiation density at an arbitrary point (x, y, z) of the plate from its base x = 0. It can be shown that the domain of integration in (4) in this case contains those points (0, y', z') of the base of the plate whose coordinates obey the inequalities

$$a^2 + y_1^2 \cot^2 \Theta_t \geqslant z_1^2, \quad a^2 + z_1^2 \cot^2 \Theta_t \geqslant y_1^2. \tag{5}$$

Here the axis Oz is perpendicular to the plane of the plate, and

$$z_1 = -z + (-1)^n z' \pm nd, \; y_1 = -y + (-1)^m y' \pm mb, \; a = x \cot \Theta_t,$$

where n and m are the numbers of reflections from the wide and narrow faces of the plate. It is clear that for a wide plate with $k_\lambda b >> 1$ the second condition in (5) must be discarded, because the reflections from the narrow faces of the plate can be neglected in this case. The general form of the domain delimited by inequalities (5) is shown in Fig. 1. The hatched zones contain points from which radiation leaves the plate and does not reach the reception point.

Going over from integration over the solid angle to integration over the domain (5) in the expression (4) for I_{f1} and making certain transformations, we obtain

$$I_{f1}(\lambda, x) = 2\pi B_\lambda(T_0) I_f^0\left(\int_0^x k_\lambda(T(x'))\,dx'\right),$$

$$I_f^0(t) = \int_1^\infty \frac{e^{-tu}}{u^2}\,du - \alpha\frac{2}{\pi}\int_n^\infty \frac{e^{-tu}}{u^2} \arccos\left(\frac{u\cos\Theta_t}{\sqrt{u^2-1}}\right) du, \tag{6}$$

$$\text{where } \alpha = \begin{cases} 1 & \text{for } k_\lambda b \gg 1, \\ 2 & \text{for } k_\lambda b \ll 1, \end{cases} \quad b/Z \ll 1.$$

An analogous expression holds of I_{f2}:

$$I_{f2}(\lambda, x) = 2\pi B_\lambda(T_1) I_f^0\left(\int_x^Z k_\lambda(T(x'))\,dx'\right). \tag{7}$$

The radiation density from the total volume of the plate I_c can be expressed in terms of I_f^0:

$$I_c(\lambda, x) = 2\pi \int_0^Z B_\lambda(T(x')) I_c^0(x, x')\,dx',$$

$$I_c^0(x, x') = \left|\frac{\partial}{\partial x'} I_f^0\left(\int_{\min(x,x')}^{\max(x,x')} k_\lambda(T(\xi))\,d\xi\right)\right|. \tag{8}$$

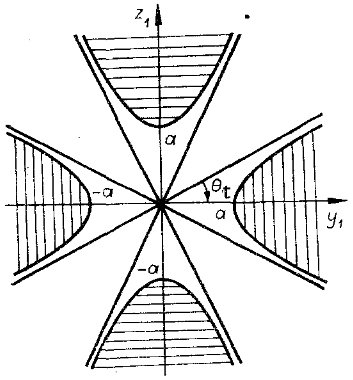

Fig. 1

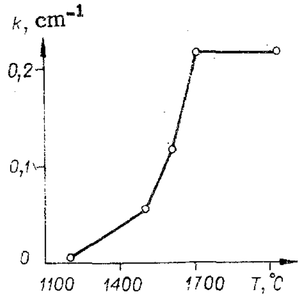

Fig. 2

To determine the shield radiation I_s, on the other hand, we invoke the assumption of a small gap between the shield and the plate. Then, reasoning as in [1], we obtain

$$I_s(\lambda, x) = 4\pi\left(1 - I_f^0(0)\right) B_\lambda(T_s). \tag{9}$$

Expressions (6)-(9) do not contain any dependence on y and z, making it superfluous to average over the plate cross section on the right-hand side of Eq. (3). It must also be noted that the radiation density I_λ does not depend on the thickness or the width of the plate.

All the concrete calculations are carried out for an infinitely wide sapphire plate for the following values of the parameters:

$$T_0 = 2326\,^\circ\mathrm{K}, \; T_1 = 1600\,^\circ\mathrm{K}, \; Z = 25\ \mathrm{cm}, \; n = 1.75.$$

As in [1], the thermal conductivity $\varkappa(T)$ is approximated by the function

$$\varkappa(T) = \begin{cases} \varkappa_0 (8.42 - 21.9T/T_0 + 16.2\,(T/T_0)^2) & \text{for} \quad T/T_0 < 0.676, \\ \varkappa_0 & \text{for} \quad T/T_0 > 0.676, \end{cases}$$

where $\varkappa_0$ = 0.01 cal/sec·cm·°K.

The temperature and frequency dependence of the absorption coefficient is plotted on the basis of [3]. For values of T lacking experimental data $k_\lambda(T)$ is calculated by linear interpolation. The frequency dependence of the absorption coefficient is approximated by a step function. The values of $k_\lambda(T)$ used in the calculations are listed in Table 1. It is important to note that at temperatures close to the melting point and for wavelengths $\lambda < 1.5$ μm the values of k_λ given in this article begin to decrease. The authors of [3] do not offer any explanation for this effect, and they express certain doubts as to the reliability of the experimental data at T = 2020°C.

Besides the selective absorption of radiation, we also consider the gray-body approximation, where the absorption coefficient depends only on the temperature. In this case the value of the absorption coefficient at temperature T is interpreted as its value $k_\lambda(T)$ at wavelength λ_{max}, as given by the relation [4]

$$n\lambda_{max}T = 0.3668\ \mathrm{cm}\cdot{}^\circ\mathrm{K}.$$

This wavelength corresponds to the radiation maximum in the absolute blackbody spectrum at this temperature. The function k(T) so obtained is shown in Fig. 2. The circle-dots represent the experimental values taken from [3]. The absorption coefficient is approximated by a linear function between them.

In the gray-body approximation Eq. (3) is simplified considerably, taking the form

$$\frac{d}{dx}\left(\varkappa(T)\frac{dT}{dx}\right) = 4k(T)\,n^2\sigma T^4 - 2k(T)\,n^2\sigma\left\{I_f^0\left(\int_0^x k(T)\,dx'\right)T_0^4 + I_f^0\left(\int_x^Z k(T)\,dx'\right)T_1^4 + \right.$$
$$\left. + \int_0^Z I_c^0(x, x')\,T^4(x')\,dx' + 2\left(1 - I_f^0(0)\right)T_s^4\right\}. \tag{10}$$

To solve the nonlinear integrodifferential equations (3) and (10) we use a conservative finite-difference scheme in conjunction with quasilinearization in each iteration. Since the

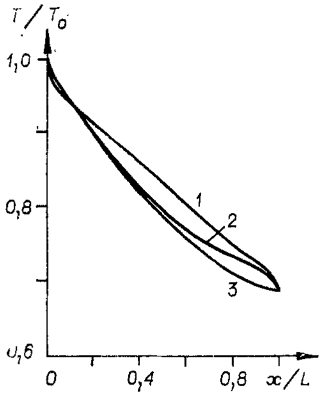

Fig. 3

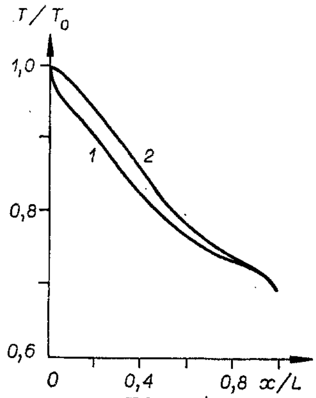

Fig. 4

kernel $I_c^0(x, x')$ of the integral operator (8) has a logarithmic singularity, and to ensure convergence of the iteration process, we transform the integral with respect to x' (8) on the right-hand side of (3) as follows:

$$\int_0^Z \int_\lambda 2\pi k_\lambda B_\lambda(T(x')) I_c^0(x, x')\, d\lambda dx' = \int_0^Z \int_\lambda 2\pi k_\lambda (B_\lambda(T(x')) - B_\lambda(T(x))) I_c^0(x, x')\, d\lambda dx' +$$
$$+ \int_\lambda 2\pi k_\lambda B_\lambda(T(x)) \left[2I_f^0(0) - I_f^0\left(\int_0^x k_\lambda d\xi\right) - I_f^0\left(\int_x^Z k_\lambda d\xi\right)\right] d\lambda.$$

Here the first term no longer has a singularity in the integrand, and the second term does not contain any integration with respect to x; it can be combined with the first term on the right-hand side of (3). We then obtain an expression of the form

$$\int_\lambda B_\lambda(T) N(x, \lambda)\, d\lambda = \frac{n^2\sigma T^4}{\pi} \int_\lambda N(x, \lambda) \left(\frac{\pi B_\lambda}{n^2\sigma T^4}\right) d\lambda. \tag{11}$$

We apply quasilinearization to the T^4 function in front of the integral and compute the remaining expressions and integrals associated with radiative energy transfer from the preceding iteration.

The shield temperature is given as a linear function:

$$T_s = T_2 - (T_2 - T_3)x/Z.$$

The results of the calculations for $T_2 = T_0$ and $T_3 = T_1$ are given in Fig. 3 for k = 0.1 cm^{-1} (curve 1) and the selective approximation (curve 3). It is seen that allowance for the temperature dependence of the absorption coefficient yields additional substantial cooling of the plate. On the other hand, the temperature distribution for selective radiation absorption in the greater part of the plate exclusive of its cold zone agrees fairly well with the temperature field in the gray-body approximation (curve 2).

The curvature of the temperature field in the vicinity of x = 0 is very pronounced and can therefore induce large thermal stresses during the pulling of crystals and significantly aggravate their defect state. Expression (9) also shows that the shields are not very effective for the heating of a semitransparent plate, because a very large part of their radiation is reflected from the surface of the plate. Consequently, to compensate for the part of the energy transported along the plate as along an optical waveguide the shields should be heated well above the melting point in the vicinity of the crystallization front. The temperature distribution in the plate with two shields is shown in Fig. 4 for the case in which the absorption coefficient depends only on the temperature: 1) T_2 = 2326°K; 2) T_2 = 2500°K. It is seen that with elevation of the temperature of the shields the variation of the temperature field near x = 0 becomes considerably less abrupt.

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NATURAL VIBRATIONAL FREQUENCIES OF A GAS OUTSIDE A CIRCULAR CYLINDRICAL SURFACE

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One of the little-studied problems in the theory of wave processes is that of natural vibrations in open regions, i.e., regions having infinitely distant points. Examples in the literature of the solution of appropriate problems are not precisely formulated. Among these, e.g., is the theory of resonators developed in the last century by Helmholtz and Rayleigh, and the theory of an open tube in acoustics [1]. Under the assumption that the process of natural vibrations in a resonator is steady, these authors estimated the effect of an opening on the frequency of vibrations, and determined the approximate degree of their damping as a consequence of the radiation of energy into the external space. They did not study the character of the vibrations of a gas far from resonance. We now assume that the vibrations of a gas can be considered steady over the whole region, clear up to infinitely distant points. Then, introducing the time dependence by the factor

$$\exp\left(-i\frac{kat}{l}\right) \qquad (k = k' + ik'',\ k' = k_0 + \Delta k,\ k'' < 0)$$

(where a is the speed of sound; l, a characteristic dimension of the resonator; k_0, reduced frequency of natural vibrations of the gas in the resonator with the opening closed; Δk, correction of the frequency introduced by the opening; and k'', a quantity characterizing the damping of the vibrations), we change over from the wave equation to the Helmholtz equation for the whole region occupied by the gas. In the absence of waves from infinity, the solution of this equation for $k'' < 0$ will increase exponentially at an infinite distance from the resonator. It obviously does not satisfy the Sommerfeld radiation conditions, and is at variance with the usual formulation of external boundary-value problems for the Helmholtz equation. Actually, of course, such a result is not realized, since the damping of free vibrations cannot continue infinitely long. However, the Helmholtz equation is a convenient model for describing vibrations of a continuous medium, and therefore a question arises of the rigorous mathematical formulation of the radiation condition for complex values of the wave number k with $k'' < 0$. It was formulated for the first time for the two-dimensional case [2], and generalized later to the three-dimensional case in [3]. It should be noted that questions related to natural vibrations in open regions arise in scattering theory. Thus, in [4] the asymptotic solution of the scattering problem outside the obstacle is written as a series in the eigenfunctions of corresponding boundary-value problems for the Helmholtz equation. In this case it was shown rigorously that the eigenfunctions satisfying the outgoing radiation condition increase exponentially at large distances from the obstacle, and the corresponding eigenvalues are complex and lie in the lower halfplane. It was shown in the three-dimensional case that the eigenvalues of the external problem for finite obstacles are discrete. Certain qualitative results concerning external eigenvalues were obtained by Arsen'ev [5, 6] who investigated the resonance properties of the solution of the scattering problem for a domain of the type of a cavity resonator. Arsen'ev showed that for a sufficiently small opening of the resonator, the poles of the solution sought are located in the neighborhood of the eigenvalues of the external and internal boundary-value problems for the respective regions without openings. In the present article we investigate the precisely formulated problem of the dependence of the complex eigenvalues of the Helmholtz equation on the size of the opening of a resonator in the form of an infinite cylinder with a longitudinal slot.

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